Image elongations in defocussed astigmatics images

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Image elongations can be generated either by tracking or guiding errors, by wind shake of the structure or by astigmatism in the wavefront. The astigmatism can be generated by small errors in the axial or radial supports, misalignments or inhomogeneous local air conditions.
The expected image elongations due to astigmatism can be estimated with a simple model which is restricted to the geometrical optics domain. The spot diagram is assumed to be made up of two contributions.

- Round random spot diagram with a, for simplicity, Gaussian distribution with a given FWHM Θ. The r.m.s. value σ_{seeing} of the distribution is then given by $\sigma_{\text{seeing}} = \Theta/(2\sqrt{\ln 2})$. and the r.m.s. of a component in one direction by $\Theta/(2\sqrt{2\ln 2})$.
- Regular elliptical spot diagram generated by a combination of third order astigmatism with a coefficient c_{ast} and defocus with a coefficient c_{def}, that is the combination $c_{\text{ast}}\, r^2 \cos 2\varphi + c_{\text{def}}\, r^2$.

The r.m.s. values of the long and short axes of the elliptical combined spot diagram are given by

$$\sigma_{\text{l}} = \sqrt{\left(\frac{\Theta}{2\sqrt{2\ln 2}}\right)^2 + \left(\frac{c_{\text{ast}} + c_{\text{def}}}{r_{\text{o}}}\right)^2} \quad (1)$$

$$\sigma_{\text{s}} = \sqrt{\left(\frac{\Theta}{2\sqrt{2\ln 2}}\right)^2 + \left(\frac{c_{\text{ast}} - c_{\text{def}}}{r_{\text{o}}}\right)^2} \quad (2)$$

Fig. 1 shows for the NTT and the VLT under seeing conditions of $\Theta = 0.4$ arcsec the ratios $E = \sigma_{\text{l}}/\sigma_{\text{s}}$ of the long to the short axis as functions of c_{def} for three different coefficients c_{ast}. In the best focus position there are no image elongations. Surprisingly, the strongest elongations do not occur for values of $c_{\text{def}} = \pm\, c_{\text{ast}}$, which correspond to the focus positions where, in a vacuum, the astigmatic images are lines. Instead, the maxima of the elongations occur for defocus values, where the r.m.s. of the spot pattern due to defocus is approximately equal to σ_{seeing}. Exactly, the coefficient of defocus, for which the maximum occurs, is given by

$$c_{\text{def,max}} = \sqrt{\left(\frac{\Theta}{2r_{\text{o}}\sqrt{2\ln 2}}\right)^2 + c_{\text{ast}}^2} \quad (3)$$

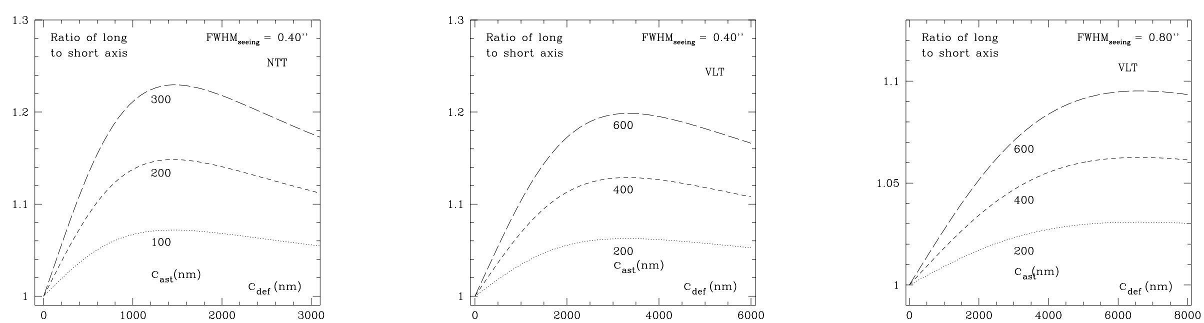

Figure 1: Image elongation as functions of the defocus for three coefficients of astigmatism. *Left*: NTT, seeing 0.4", *Middle*: VLT, seeing 0.4", *Right*: VLT, seeing 0.8".

At the NTT, with a seeing of $\Theta = 0.40$ arcsec the combination of modest amounts of astigmatism with a coefficient of $c_{\text{ast}} = 300$ nm and defocus with a coefficient of $c_{\text{def}} = 200$ nm generate generate image elongation of the order of 10%. Through focus sequences taken at the NTT, where the coefficient of astigmatism was of the order of 300 nm confirmed the order of magnitude of the estimated elongations.
At the VLT the r.m.s. of c_{ast} and c_{def} during operation with closed loop corrections are, mainly due to local air effects, of the order of 600 nm. With seeing values of $\Theta = 0.4$ arcsec the image elongation will then, on average, be of the order of 6%, and with seeing values of $\Theta = 0.8$ arcsec of the order of 1.5%.
The noise level for the detection of image elongation is roughly of the order of 2 to 3%. The lower value should then be the limit 2% for the permissible image elongations generated by a combination of astigmatism and defocus. Assuming that defocus and astigmatism can be controlled with the same accuracy, the limits for the coefficients of astigmatism and defocus are then at the VLT of the order of 300 nm for a seeing of 0.40 arcsec and of the order of 700 nm for a seeing of 0.80 arcsec.